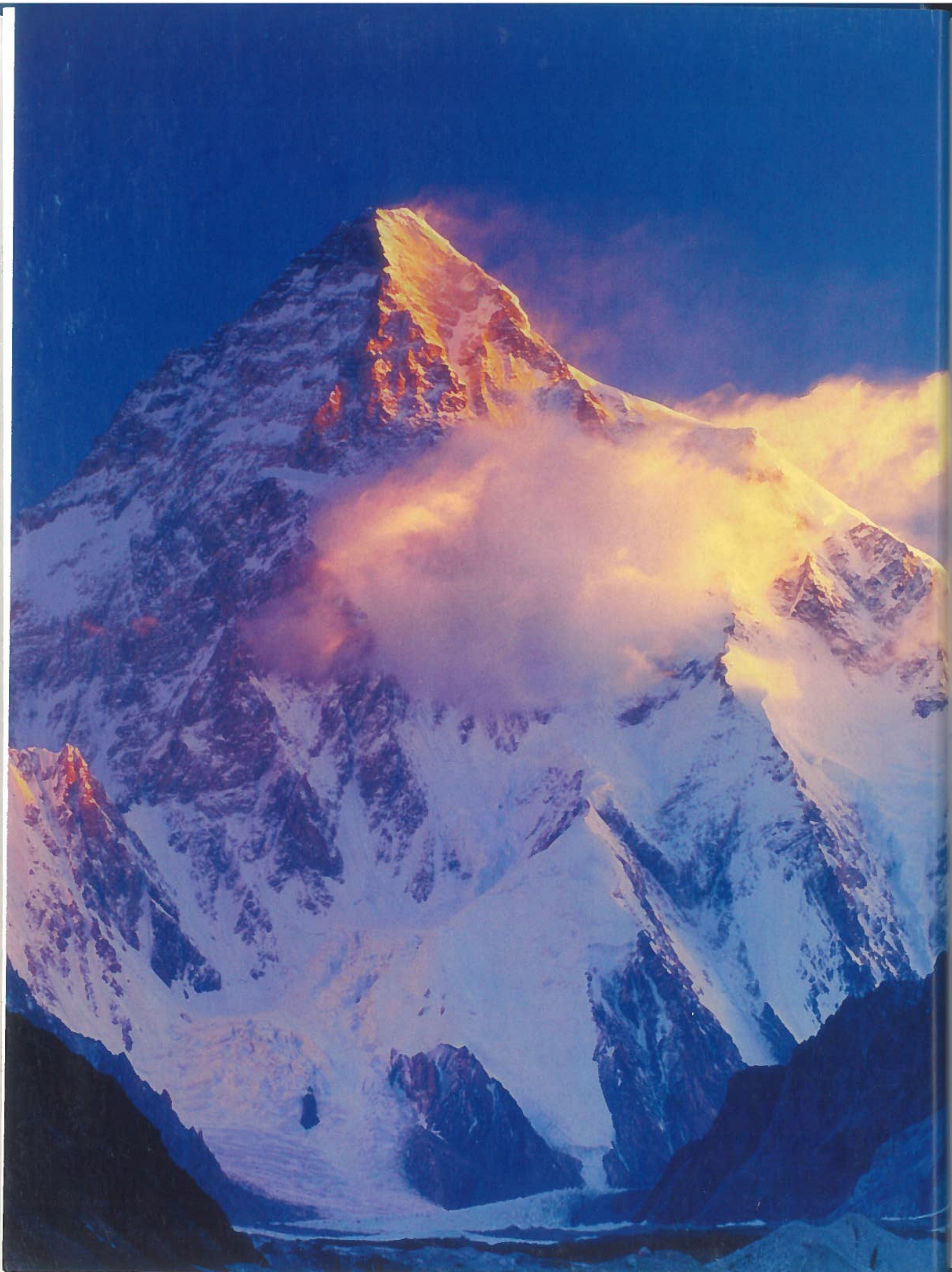

PHYSICS

PRINCIPLES WITH APPLICATIONS

SIXTH EDITION

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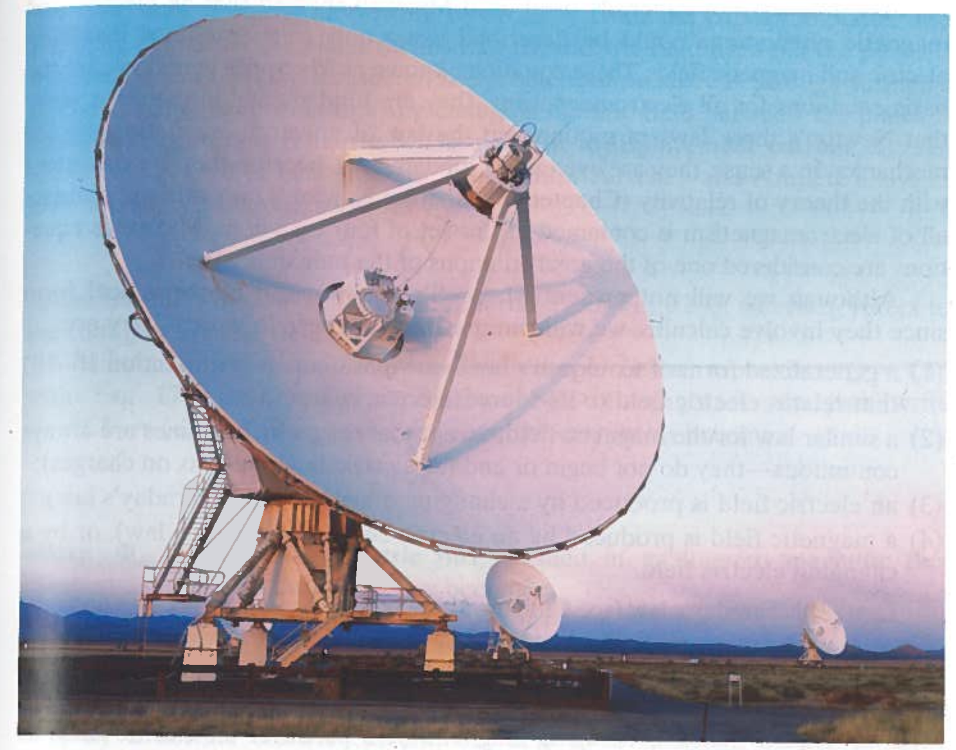


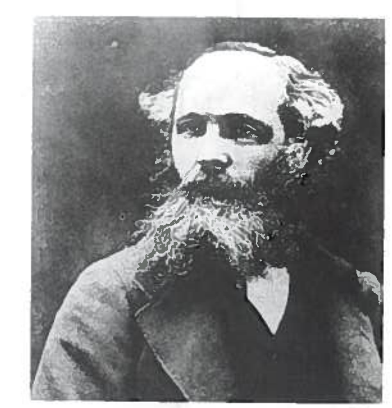
These circular disk antennas, each 25 m in diameter, are pointed to receive radio waves from out in space. Radio waves are electromagnetic (EM) waves that have frequencies from a few hundred Hz to about 100 MHz. These antennas are connected together electronically to achieve better detail, and are a part of the Very Large Array in New Mexico searching the heavens for information about the Cosmos.

Maxwell predicted the existence of EM waves from his famous equations, which are a magnificent summary of electromagnetism.

CHAPTER 22

Electromagnetic Waves

FIGURE 22–1 James Clerk Maxwell.



The culmination of electromagnetic theory in the nineteenth century was the prediction, and the experimental verification, that waves of electromagnetic fields could travel through space. This achievement opened a whole new world of communication: first the wireless telegraph, then radio and television, and more recently cell phones and remote-control devices. And it yielded the spectacular prediction that light is an electromagnetic wave.

The theoretical prediction of electromagnetic waves was the work of the Scottish physicist James Clerk Maxwell (1831–1879; Fig. 22–1), who unified, in one magnificent theory, all the phenomena of electricity and magnetism.

22–1 Changing Electric Fields Produce Magnetic Fields; Maxwell's Equations

The development of electromagnetic theory in the early part of the nineteenth century by Oersted, Ampère, and others was not actually done in terms of electric and magnetic fields. The idea of the field was introduced somewhat later by Faraday, and was not generally used until Maxwell showed that all electric and magnetic phenomena could be described using only four equations involving electric and magnetic fields. These equations, known as **Maxwell's equations**, are the basic equations for all electromagnetism. They are fundamental in the same sense that Newton's three laws of motion and the law of universal gravitation are for mechanics. In a sense, they are even more fundamental, because they are consistent with the theory of relativity (Chapter 26), whereas Newton's laws are not. Because all of electromagnetism is contained in this set of four equations, Maxwell's equations are considered one of the great triumphs of the human intellect.

Although we will not present Maxwell's equations in mathematical form since they involve calculus, we will summarize them here in words. They are:

Maxwell's equations

(1) a generalized form of Coulomb's law known as Gauss's law (Section 16–10) that relates electric field to its source, electric charge;
(2) a similar law for the magnetic field, except that magnetic field lines are always continuous—they do not begin or end (as electric field lines do, on charges);
(3) an electric field is produced by a changing magnetic field (Faraday's law);
(4) a magnetic field is produced by an electric current (Ampère's law), or by a changing electric field.

Law (3) is Faraday's law (see Chapter 21, especially Section 21–4). The first part of law (4), that a magnetic field is produced by an electric current, was discovered by Oersted, and the mathematical relation is given by Ampère's law (Section 20–8). But the second part of law (4) is an entirely new aspect predicted by Maxwell. Maxwell argued that if a changing magnetic field produces an electric field, as given by Faraday's law, then the reverse might be true as well: **a changing electric field will produce a magnetic field.** This was an *hypothesis* by Maxwell, based on the idea of symmetry in nature. Indeed, the size of the effect in most cases is so small that Maxwell recognized it would be difficult to detect it experimentally.

Changing $\vec{\mathbf{E}}$ produces $\vec{\mathbf{B}}$

* Maxwell's Fourth Equation (Ampère's Law Extended)

To back up the idea that a changing electric field might produce a magnetic field, we use an indirect argument that goes something like this. According to Ampère's law (Section 20–8), $\Sigma B_{\parallel}\,\Delta l = \mu_0 I$. That is, divide any closed path you choose into short segments Δl, multiply each segment by the parallel component of the magnetic field B at that segment, and then sum all these products over the complete closed path. That sum will then equal μ_0 times the total current I that passes through a surface bounded by the path. When we applied Ampère's law to the field around a straight wire (Section 20–8), we imagined the current as passing through the circular area enclosed by our circular loop. That area is the flat surface 1 shown in Fig. 22–2. However, we could just as well use the sack-shaped surface 2 in Fig. 22–2 as the surface for Ampère's law because the same current I passes through it.

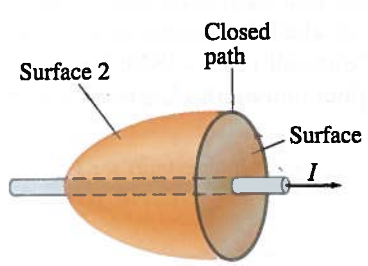


FIGURE 22–2 Ampère's law applied to two different surfaces bounded by the same closed path.

Now consider the closed path for the situation of Fig. 22–3, where a capacitor is being discharged. Ampère's law works for surface 1 (current I passes through surface 1), but it does not work for surface 2 because no current passes through surface 2. There is a magnetic field around the wire, so the left side of Ampère's law is not zero around the circular closed path; yet no current flows through surface 2, so the right side *is* zero for surface 2. We seem to have a contradiction of Ampère's law. There is a magnetic field present in Fig. 22–3, however, only if charge is flowing to or away from the capacitor plates. The changing charge on the plates means that the electric field between the plates is changing in time. Maxwell resolved the problem of no current through surface 2 in Fig. 22–3 by proposing that the changing electric field between the plates is *equivalent to* an electric current. He called it a **displacement current**, I_D. An ordinary current I is then called a "conduction current," and Ampère's law, as generalized by Maxwell, becomes

$$\Sigma B_{\parallel}\,\Delta l = \mu_0(I + I_D).$$

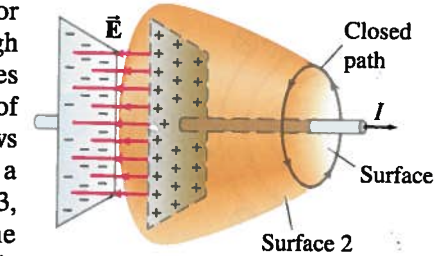


FIGURE 22–3 A capacitor discharging. No conduction current passes through surface 2. An extra term is needed in Ampère's law.

Ampère's law will now apply also for surface 2 in Fig. 22–3, where I_D refers to the changing electric field.

By combining Eq. 17–7 for the charge on a capacitor, $Q = CV$, with Eq. 17–4a, $V = Ed$, and Eq. 17–8, $C = \epsilon_0 A/d$, we can write $Q = CV = (\epsilon_0 A/d)(Ed) = \epsilon_0 AE$. Then the current I_D becomes

$$I_D = \frac{\Delta Q}{\Delta t} = \epsilon_0 \frac{\Delta \Phi_E}{\Delta t},$$

where $\Phi_E = EA$ is the **electric flux**, defined in analogy to magnetic flux (Section 21–2). Then, Ampère's law becomes

Ampère's law (generalized)

$$\Sigma B_{\parallel}\,\Delta l = \mu_0 I + \mu_0\epsilon_0 \frac{\Delta \Phi_E}{\Delta t}. \qquad \textbf{(22–1)}$$

This equation embodies Maxwell's idea that a magnetic field can be caused not only by a normal electric current, but also by a changing electric field or changing electric flux.

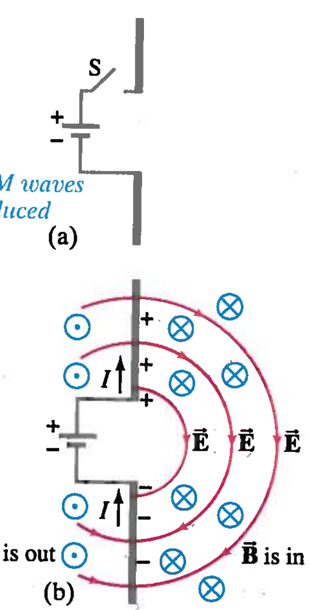
22–2 Production of Electromagnetic Waves

According to Maxwell, a magnetic field will be produced in empty space if there is a changing electric field. From this, Maxwell derived another startling conclusion. If a changing magnetic field produces an electric field, that electric field is itself changing. This changing electric field will, in turn, produce a magnetic field, which will be changing, and so it too will produce a changing electric field; and so on. When Maxwell worked with his equations, he found that the net result of these interacting changing fields was a *wave* of electric and magnetic fields that can propagate (travel) through space! We now examine, in a simplified way, how such **electromagnetic waves** can be produced.

How EM waves are produced

Consider two conducting rods that will serve as an "antenna" (Fig. 22–4a). Suppose that these two rods are connected by a switch to the opposite terminals of a battery. As soon as the switch is closed, the upper rod quickly becomes positively charged and the lower one negatively charged. Electric field lines are formed as indicated by the lines in Fig. 22–4b. While the charges are flowing, a current exists whose direction is indicated by the black arrows. A magnetic field is therefore produced near the antenna. The magnetic field lines encircle the rod-like antenna and therefore, in Fig. 22–4, $\vec{\mathbf{B}}$ points into the page (⊗) on the right and out of the page (⊙) on the left. Now we ask, how far out do these electric and magnetic fields extend? In the static case, the fields extend outward indefinitely far. However, when the switch in Fig. 22–4 is closed, the fields quickly appear nearby, but it takes time for them to reach distant points. Both electric and magnetic fields store energy, and this energy cannot be transferred to distant points at infinite speed.

FIGURE 22–4 Fields produced by charge flowing into conductors. It takes time for the $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields to travel outward to distant points. The fields are shown to the right of the antenna, but they move out in all directions, symmetrically about the (vertical) antenna.



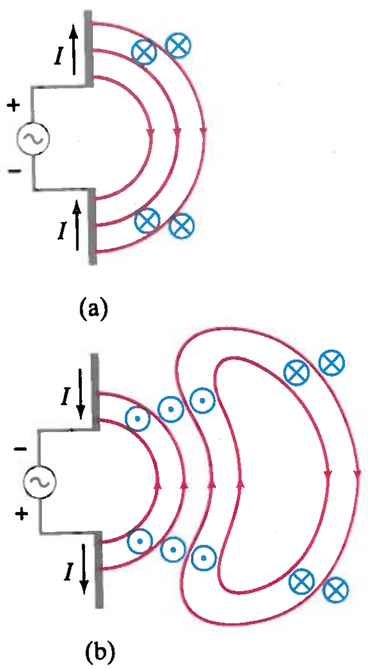
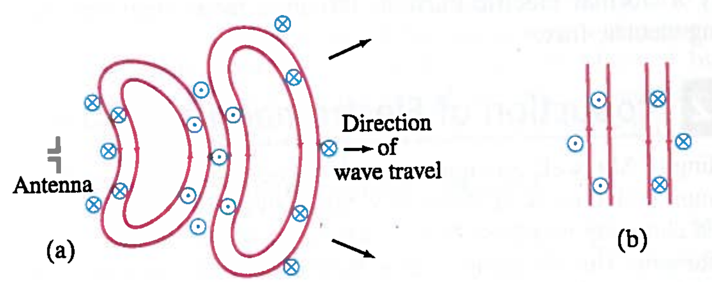


FIGURE 22–5 Sequence showing electric and magnetic fields that spread outward from oscillating charges on two conductors (the antenna) connected to an ac source (see the text).

Now we look at the situation of Fig. 22–5, where our antenna is connected to an ac generator. In Fig. 22–5a, the connection has just been completed. Charge starts building up, and fields form just as in Fig. 22–4b. The + and − signs in Fig. 22–5a indicate the net charge on each rod. The black arrows indicate the direction of the current. The electric field is represented by red lines in the plane of the page; and the magnetic field, according to the right-hand rule, is into (⊗) or out of (⊙) the page. In Fig. 22–5b, the voltage of the ac generator has reversed in direction; the current is reversed and the new magnetic field is in the opposite direction. Because the new fields have changed direction, the old lines fold back to connect up to some of the new lines and form closed loops as shown.† The old fields, however, don't suddenly disappear; they are on their way to distant points. Indeed, because a changing magnetic field produces an electric field, and a changing electric field produces a magnetic field, this combination of changing electric and magnetic fields moving outward is self-supporting, no longer depending on the antenna charges.

The fields not far from the antenna, referred to as the *near field*, become quite complicated, but we are not so interested in them. We are mainly interested in the fields far from the antenna (they are generally what we detect), which we refer to as the **radiation field**. The electric field lines form loops, as shown in Fig. 22–6, and continue moving outward. The magnetic field lines also form closed loops, but are not shown since they are perpendicular to the page. Although the lines are shown only on the right of the source, fields also travel in other directions. The field strengths are greatest in directions perpendicular to the oscillating charges; and they drop to zero along the direction of oscillation—above and below the antenna in Fig. 22–6.

FIGURE 22–6 (a) The radiation fields (far from the antenna) produced by a sinusoidal signal on the antenna. The red closed loops represent electric field lines. The magnetic field lines, perpendicular to the page and represented by blue ⊗ and ⊙, also form closed loops. (b) Very far from the antenna, the wave fronts (field lines) are essentially flat over a fairly large area, and are referred to as *plane waves*.



The magnitudes of both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ in the radiation field are found to decrease with distance as $1/r$. (Compare this to the static electric field given by Coulomb's law where $\vec{\mathbf{E}}$ decreases as $1/r^2$.) The energy carried by the electromagnetic wave is proportional (as for any wave, Chapter 11) to the square of the amplitude, E^2 or B^2, as will be discussed further in Section 22–7, so the intensity of the wave decreases as $1/r^2$.

$\vec{\mathbf{E}} \perp \vec{\mathbf{B}} \perp \vec{\mathbf{v}}$ in EM wave

Several things about the radiation field can be noted from Fig. 22–6. First, *the electric and magnetic fields at any point are perpendicular to each other, and to the direction of wave travel.* Second, we can see that the fields alternate in direction ($\vec{\mathbf{B}}$ is into the page at some points and out of the page at others; $\vec{\mathbf{E}}$ points up at some points and down at others). Thus, the field strengths vary from a maximum in one direction, to zero, to a maximum in the other direction. The electric and magnetic fields are "in phase": that is, they each are zero at the same points and reach their maxima at the same points in space. Finally, very far from the antenna (Fig. 22–6b) the field lines are quite flat over a reasonably large area, and the waves are referred to as **plane waves**.

†We are considering waves traveling through empty space. There are no charges for lines of $\vec{\mathbf{E}}$ to start or stop on, so they form closed loops. Magnetic field lines always form closed loops.

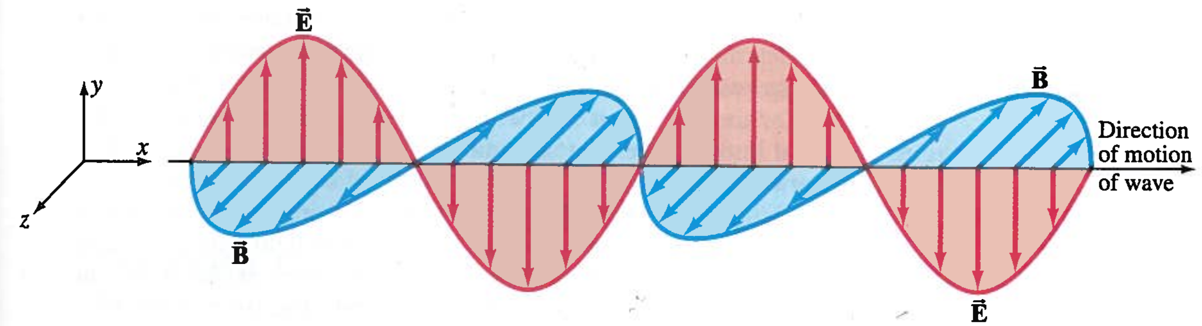


FIGURE 22–7 Electric and magnetic field strengths in an electromagnetic wave. $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are at right angles to each other. The entire pattern moves in a direction perpendicular to both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$.

If the source voltage varies sinusoidally, then the electric and magnetic field strengths in the radiation field will also vary sinusoidally. The sinusoidal character of the waves is shown in Fig. 22–7, which displays the field *strengths* as a function of position along the direction of wave travel. Notice that $\vec{\mathbf{B}}$ and $\vec{\mathbf{E}}$ are perpendicular to each other and to the direction of wave travel.

We call these waves electromagnetic (EM) waves. They are *transverse* waves because the amplitude is perpendicular to the direction of wave travel. However, EM waves are always waves of *fields*, not of matter (like waves on water or a rope). Because they are fields, EM waves can propagate in empty space.

As we have seen, EM waves are produced by electric charges that are oscillating, and hence are undergoing acceleration. In fact, we can say in general that

EM waves are produced by accelerating electric charges

accelerating electric charges give rise to electromagnetic waves.

Maxwell derived a formula for the speed of EM waves:

$$v = c = \frac{E}{B}, \qquad \text{(22–2)}$$

c is symbol for speed of light

where c is the special symbol for the speed of electromagnetic waves in empty space, and E and B are the magnitudes of electric and magnetic fields at the same point in space. More specifically, it was easily shown also that

Speed of EM waves

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}. \qquad \text{(22–3)}$$

When Maxwell put in the values for ϵ_0 and μ_0, he found

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(4\pi \times 10^{-7}\,\mathrm{N\cdot s^2/C^2})}}$$
$$= 3.00 \times 10^8\,\mathrm{m/s},$$

which is equal to the measured speed of light in vacuum.

22–3 Light as an Electromagnetic Wave and the Electromagnetic Spectrum

Maxwell's prediction that EM waves should exist was startling. Equally remarkable was the speed at which EM waves were predicted to travel—$3.00 \times 10^8\,\mathrm{m/s}$, the same as the measured speed of light.

Light had been shown some 60 years before Maxwell's work to behave like a wave (we'll discuss this in Chapter 24). But nobody knew what kind of wave it was. What is it that is oscillating in a light wave? Maxwell, on the basis of the calculated speed of EM waves, argued that light must be an electromagnetic wave. This idea soon came to be generally accepted by scientists, but not fully until after EM waves were experimentally detected. EM waves were first generated and detected experimentally by Heinrich Hertz (1857–1894) in 1887, eight years after Maxwell's death. Hertz used a spark-gap apparatus in which charge was made to rush back and forth for a short time, generating waves whose frequency was about 10^9 Hz. He detected them some distance away using a loop of wire in which an emf was produced when a changing magnetic field passed through. These waves were later shown to travel at the speed of light, 3.00×10^8 m/s, and to exhibit all the characteristics of light such as reflection, refraction, and interference. The only difference was that they were not visible. Hertz's experiment was a strong confirmation of Maxwell's theory.

The wavelengths of visible light were measured in the first decade of the nineteenth century, long before anyone imagined that light was an electromagnetic wave. The wavelengths were found to lie between 4.0×10^{-7} m and 7.5×10^{-7} m; or 400 nm to 750 nm (1 nm = 10^{-9} m). The frequencies of visible light can be found using Eq. 11–12, which we rewrite here:

Wavelength and frequency related to speed

$$c = \lambda f, \tag{22–4}$$

where f and λ are the frequency and wavelength, respectively, of the wave. Here, c is the speed of light, 3.00×10^8 m/s; it gets the special symbol c because of its universality for all EM waves in free space. Equation 22–4 tells us that the frequencies of visible light are between 4.0×10^{14} Hz and 7.5×10^{14} Hz. (Recall that 1 Hz = 1 cycle per second = 1 s^{-1}.)

But visible light is only one kind of EM wave. As we have seen, Hertz produced EM waves of much lower frequency, about 10^9 Hz. These are now called **radio waves**, since frequencies in this range are used to transmit radio and TV signals. Electromagnetic waves, or EM radiation as we sometimes call it, have been produced or detected over a wide range of frequencies. They are usually categorized as shown in Fig. 22–8, which is known as the **electromagnetic spectrum.**

EM spectrum

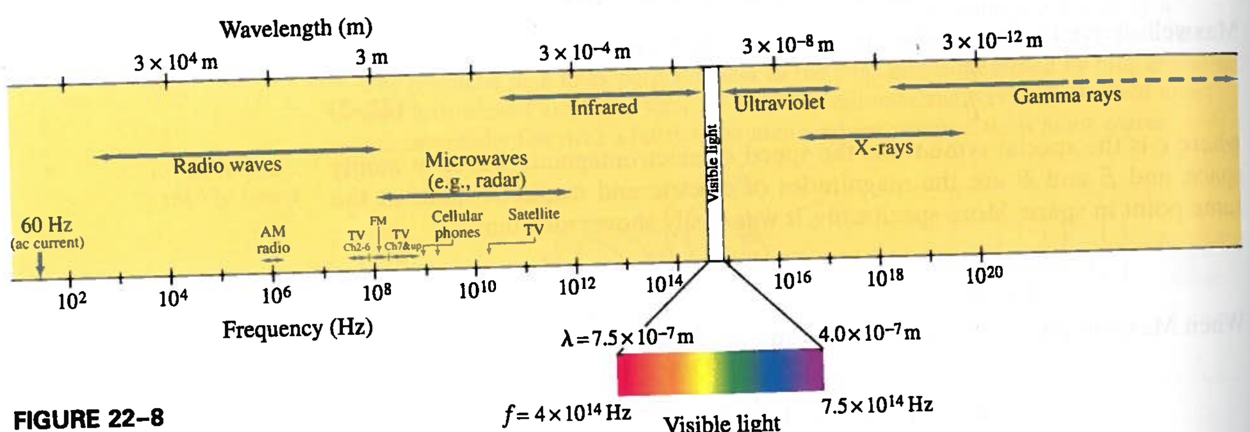


FIGURE 22–8 Electromagnetic spectrum.

Radio waves and microwaves can be produced in the laboratory using electronic equipment (Fig. 22–5). Higher-frequency waves are very difficult to produce electronically. These and other types of EM waves are produced in natural processes, as emission from atoms, molecules, and nuclei (more on this later). EM waves can be produced by the acceleration of electrons or other charged particles, such as electrons accelerating in the antenna of Fig. 22–5. Another example is X-rays, which are produced (Chapters 25 and 28) when fast-moving electrons are rapidly decelerated upon striking a metal target. Even the visible light emitted by an ordinary incandescent bulb is due to electrons undergoing acceleration within the hot filament.

We will meet various types of EM waves later. However, it is worth mentioning here that infrared (IR) radiation (EM waves whose frequency is just less than that of visible light) is mainly responsible for the heating effect of the Sun. The Sun emits not only visible light but substantial amounts of IR and UV (ultraviolet) as well. The molecules of our skin tend to "resonate" at infrared frequencies, so it is these that are preferentially absorbed and thus warm us. We humans experience EM waves differently depending on their wavelengths: Our eyes detect wavelengths between about 4×10^{-7} m and 7.5×10^{-7} m (visible light), whereas our skin detects longer wavelengths (IR). Many EM wavelengths we don't detect directly at all.

Light and other electromagnetic waves travel at a speed of 3×10^8 m/s. Compare this to sound, which travels (see Chapter 12) at a speed of about 300 m/s in air, a million times slower; or to typical freeway speeds of a car, 30 m/s (100 km/h, or 60 mi/h), 10 million times slower than light. EM waves differ from sound waves in another big way: sound waves travel in a medium such as air, and involve motion of air molecules; EM waves do not involve any material—only fields, and they can travel in empty space.

CAUTION
Sound and EM waves are different

EXAMPLE 22–1 Wavelengths of EM waves. Calculate the wavelength (*a*) of a 60-Hz EM wave, (*b*) of a 93.3-MHz FM radio wave, and (*c*) of a beam of visible red light from a laser at frequency 4.74×10^{14} Hz.

APPROACH All of these waves are electromagnetic waves, so their speed is $c = 3.00 \times 10^8$ m/s. We solve for λ in Eq. 22–4: $\lambda = c/f$.

SOLUTION (*a*)

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\,\text{m/s}}{60\,\text{s}^{-1}} = 5.0 \times 10^6\,\text{m},$$

or 5000 km. 60 Hz is the frequency of ac current in the United States, and, as we see here, one wavelength stretches all the way across the continental USA.
(*b*)

$$\lambda = \frac{3.00 \times 10^8\,\text{m/s}}{93.3 \times 10^6\,\text{s}^{-1}} = 3.22\,\text{m}.$$

The length of an FM antenna is about half this ($\frac{1}{2}\lambda$), or $1\frac{1}{2}$ m.
(*c*)

$$\lambda = \frac{3.00 \times 10^8\,\text{m/s}}{4.74 \times 10^{14}\,\text{s}^{-1}} = 6.33 \times 10^{-7}\,\text{m}\ (= 633\,\text{nm}).$$

EXERCISE A What are the frequencies of (*a*) an 80-m-wavelength radio wave, and (*b*) an X-ray of wavelength 5.5×10^{-11} m?

EXAMPLE 22–2 ESTIMATE Cell phone antenna. The antenna of a cell phone is often $\frac{1}{4}$ wavelength long. A particular cell phone has an 8.5-cm-long straight rod for its antenna. Estimate the operating frequency of this phone.

APPROACH The basic equation relating wave speed, wavelength, and frequency is $c = \lambda f$; the wavelength λ equals four times the antenna's length.

SOLUTION The antenna is $\frac{1}{4}\lambda$ long, so $\lambda = 4(8.5\,\text{cm}) = 34\,\text{cm} = 0.34\,\text{m}$. Then $f = c/\lambda = (3.0 \times 10^8\,\text{m/s})/(0.34\,\text{m}) = 8.8 \times 10^8\,\text{Hz} = 880\,\text{MHz}$.

NOTE Radio antennas are not always straight conductors. The conductor may be a round loop to save space. See Fig. 22–17b.

EXERCISE B How long should a $\frac{1}{4}$-λ antenna be for an aircraft radio operating at 165 MHz?

Electromagnetic waves can travel along transmission lines as well as in empty space. When a source of emf is connected to a transmission line—be it two parallel wires or a coaxial cable (Fig. 22–9)—the electric field within the wire is not set up immediately at all points along the wires. This is based on the same argument we used in Section 22–2 with reference to Fig. 22–5. Indeed, it can be shown that if the wires are separated by empty space or air, the electrical signal travels along the wires at the speed $c = 3.0 \times 10^8$ m/s. For example, when you flip a light switch, the light actually goes on a tiny fraction of a second later. If the wires are in a medium whose electric permittivity is ϵ and magnetic permeability is μ (Sections 17–8 and 20–12, respectively), the speed is not given by Eq. 22–3, but by

$$v = \frac{1}{\sqrt{\epsilon\mu}}.$$

FIGURE 22–9 Coaxial cable.

EXAMPLE 22–3 ESTIMATE Voice speed through the wires. When you speak on the telephone from Los Angeles to a friend in New York some 4000 km away, how long does it take the signal carrying your voice to travel that distance?

APPROACH The signal is carried on a telephone wire or in the air via satellite. In either case it is an electromagnetic wave. Electronics as well as the wire or cable slow things down, but as a rough estimate we take the speed to be $c = 3.0 \times 10^8\,\mathrm{m/s}$.

SOLUTION Since speed = distance/time, then time = distance/speed = $(4.0 \times 10^6\,\mathrm{m})/(3.0 \times 10^8\,\mathrm{m/s}) = 1.3 \times 10^{-2}\,\mathrm{s}$, or about $\frac{1}{100}$ s.

NOTE Such a small amount of time normally goes unnoticed.

EXERCISE C If your voice traveled as a sound wave, how long would it take in Example 22–3?

22–4 Measuring the Speed of Light

Galileo attempted to measure the speed of light by trying to measure the time required for light to travel a known distance between two hilltops. He stationed an assistant on one hilltop and himself on another, and ordered the assistant to lift the cover from a lamp the instant he saw a flash from Galileo's lamp. Galileo measured the time between the flash of his lamp and when he received the light from his assistant's lamp. The time was so short that Galileo concluded it merely represented human reaction time, and that the speed of light must be extremely high.

The first successful determination that the speed of light is finite was made by the Danish astronomer Ole Roemer (1644–1710). Roemer had noted that the carefully measured orbital period of Io, a moon of Jupiter with an average period of 42.5 h, varied slightly, depending on the relative motion of Earth and Jupiter. When Earth was moving away from Jupiter, the period of Io was slightly longer, and when Earth was moving toward Jupiter, the period was slightly shorter. He attributed this variation in the apparent period to the change in distance between the Earth and Jupiter during one of Io's periods, and the time it took light to travel this distance. Roemer concluded that the speed of light—though great—is finite.

Michelson measures c

Since then a number of techniques have been used to measure the speed of light. Among the most important were those carried out by the American Albert A. Michelson (1852–1931). Michelson used the rotating mirror apparatus diagrammed in Fig. 22–10 for a series of high-precision experiments carried out from 1880 to the 1920s. Light from a source was directed at one face of a rotating eight-sided mirror. The reflected light traveled to a stationary mirror a large distance away and back again as shown. If the rotating mirror was turning at just the right rate, the returning beam of light would reflect from one face of the mirror into a small telescope through which the observer looked. If the speed of rotation was only slightly different, the beam would be deflected to one side and would not be seen by the observer. From the required speed of the rotating mirror and the known distance to the stationary mirror, the speed of light could be calculated. In the 1920s, Michelson set up the rotating mirror on the top of Mt. Wilson in southern California and the stationary mirror on Mt. Baldy (Mt. San Antonio) 35 km away. He later measured the speed of light in vacuum using a long evacuated tube.

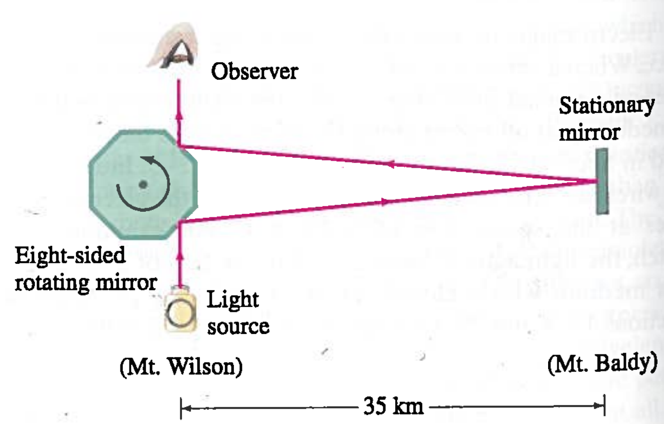


FIGURE 22–10 Michelson's speed-of-light apparatus (not to scale).

Today the speed of light, c, in vacuum is taken as

$$c = 2.99792458 \times 10^8\,\mathrm{m/s},$$

and is defined to be this value. This means that the standard for length, the meter, is no longer defined separately. Instead, as we noted in Section 1–5, the meter is now formally defined as the distance light travels in vacuum in 1/299,792,458 of a second.

We usually round off c to

$$c = 3.00 \times 10^8\,\mathrm{m/s}$$

when extremely precise results are not required. In air, the speed is only slightly less.

*22–5 Energy in EM Waves

Electromagnetic waves carry energy from one region of space to another. This energy is associated with the moving electric and magnetic fields. In Section 17–9, we saw that the energy density u_E (J/m³) stored in an electric field E is $u_E = \frac{1}{2}\epsilon_0 E^2$ (Eq. 17–11). The energy density stored in a magnetic field B, as we discussed in Section 21–10, is given by $u_B = \frac{1}{2}B^2/\mu_0$ (Eq. 21–10). Thus, the total energy stored per unit volume in a region of space where there is an electromagnetic wave is

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}. \tag{22–5}$$

In this equation, E and B represent the electric and magnetic field strengths of the wave at any instant in a small region of space. We can write Eq. 22–5 in terms of the E field only using Eqs. 22–2 ($B = E/c$) and 22–3 ($c = 1/\sqrt{\epsilon_0\mu_0}$) to obtain

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{\epsilon_0\mu_0 E^2}{\mu_0} = \epsilon_0 E^2. \tag{22–6a}$$

Note here that the energy density associated with the B field equals that due to the E field, and each contributes half to the total energy. We can also write the energy density in terms of the B field only:

$$u = \epsilon_0 E^2 = \epsilon_0 c^2 B^2 = \frac{B^2}{\mu_0}, \tag{22–6b}$$

or in one term containing both E and B,

$$u = \epsilon_0 E^2 = \epsilon_0 EcB = \frac{\epsilon_0 EB}{\sqrt{\epsilon_0\mu_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}}\,EB. \tag{22–6c}$$

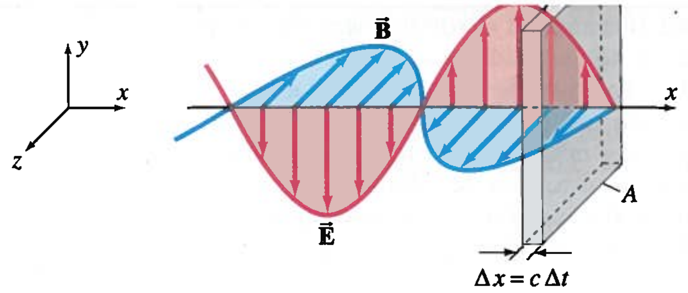


FIGURE 22–11 Electromagnetic wave carrying energy through area A.

The energy a wave transports per unit time per unit area is the **intensity** I, as defined in Sections 11–9 and 12–2.† The units of I are W/m². The energy passing through an area A in a time Δt (see Fig. 22–11) is

$$\Delta U = u\,\Delta V = (u)(A\,\Delta x) = (\epsilon_0 E^2)(Ac\,\Delta t)$$

because $\Delta x = c\,\Delta t$. Therefore, the magnitude of the intensity (energy per unit area per time Δt, or power per unit area) is

$$I = \frac{\Delta U}{A\,\Delta t} = \frac{(\epsilon_0 E^2)(Ac\,\Delta t)}{A\,\Delta t} = \epsilon_0 c E^2.$$

From Eqs. 22–2 and 22–3, this can also be written

Intensity of EM waves

$$I = \epsilon_0 c E^2 = \frac{c}{\mu_0} B^2 = \frac{EB}{\mu_0}. \tag{22–7}$$

The *average intensity* over an extended period of time, if E and B are sinusoidal so that $\overline{E^2} = E_0^2/2$ (just as for electric currents and voltages, Section 18–7), is

Average intensity

$$\overline{I} = \frac{1}{2}\epsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2\mu_0}. \tag{22–8}$$

Here E_0 and B_0 are the maximum values of E and B. We can also write

$$\overline{I} = \frac{E_{\text{rms}} B_{\text{rms}}}{\mu_0},$$

where E_{rms} and B_{rms} are the rms values $\left(E_{\text{rms}} = \sqrt{\overline{E^2}},\ B_{\text{rms}} = \sqrt{\overline{B^2}}\right)$.

EXAMPLE 22–4 ***E* and *B* from the Sun.** Radiation from the Sun reaches the Earth (above the atmosphere) at a rate of about $1350\,\mathrm{J/s \cdot m^2}$ $(=1350\,\mathrm{W/m^2})$. Assume that this is a single EM wave, and calculate the maximum values of E and B.

APPROACH We are given the intensity $\overline{I} = 1350\,\mathrm{J/s \cdot m^2}$. We solve Eq. 22–8 $\left(\overline{I} = \frac{1}{2}\epsilon_0 c E_0^2\right)$ for E_0 in terms of $\overline{I}$.

SOLUTION
$$E_0 = \sqrt{\frac{2\overline{I}}{\epsilon_0 c}} = \sqrt{\frac{2(1350\,\mathrm{J/s\cdot m^2})}{(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(3.00 \times 10^8\,\mathrm{m/s})}}$$
$$= 1.01 \times 10^3\,\mathrm{V/m}.$$

From Eq. 22–2, $B = E/c$, so

$$B_0 = \frac{E_0}{c} = \frac{1.01 \times 10^3\,\mathrm{V/m}}{3.00 \times 10^8\,\mathrm{m/s}} = 3.37 \times 10^{-6}\,\mathrm{T}.$$

NOTE Although B has a small numerical value compared to E (because of the way the different units for E and B are defined), B contributes the same energy to the wave as E does, as we saw earlier.

†The intensity I for EM waves is often called the **Poynting vector** and given the symbol $\vec{\mathbf{S}}$. Its direction is that in which the energy is being transported, which is the direction the wave is traveling, and its magnitude is the intensity $(S = I)$.